Introduction to Graphs

Graphs!

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Euler

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- Euler
- Definitions: model.

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- Euler Again!!

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  Definitions: model.
  Euler Again!!

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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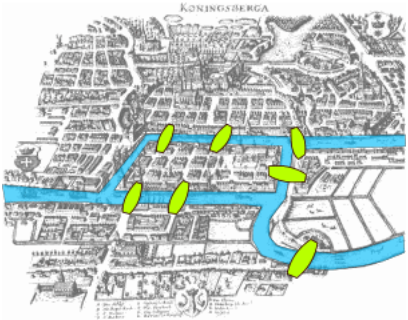

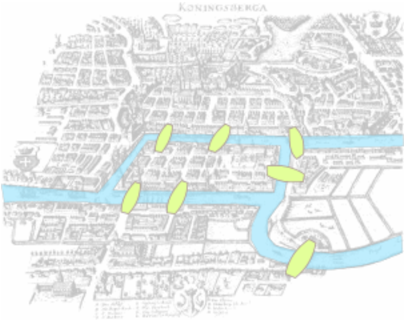

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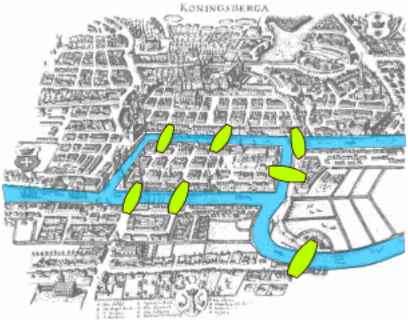

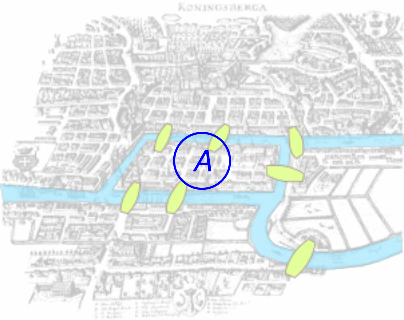

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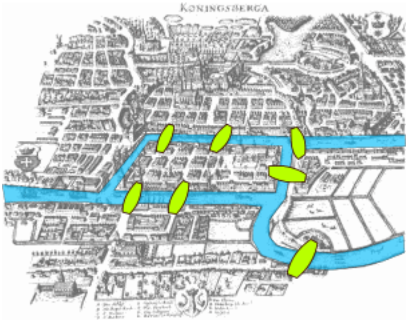

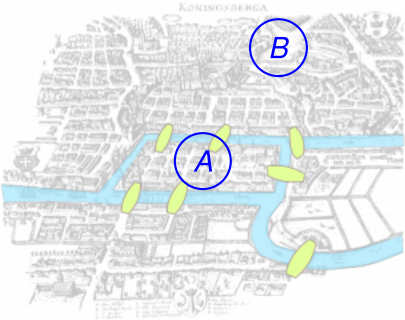

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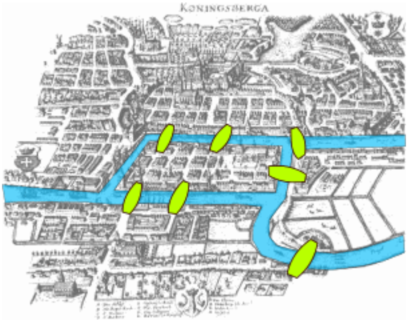

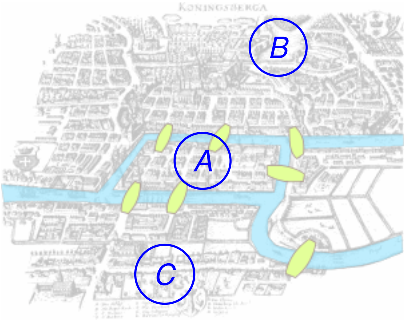

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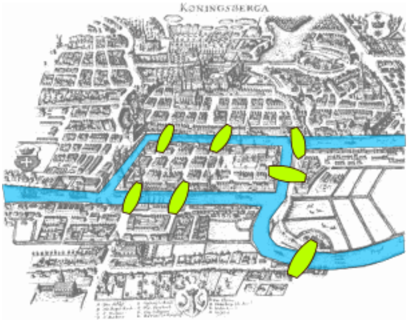

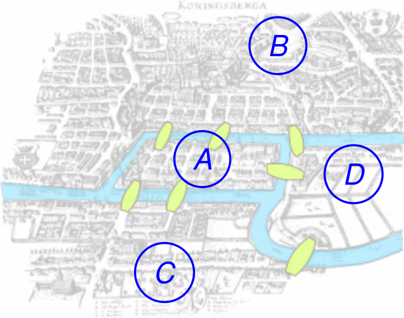

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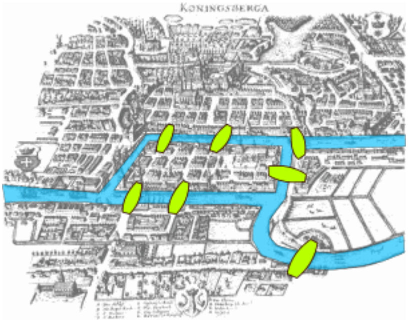

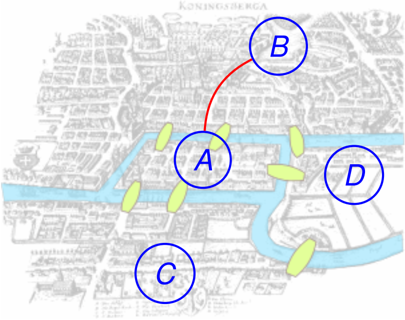

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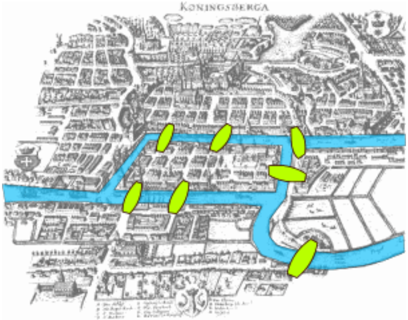

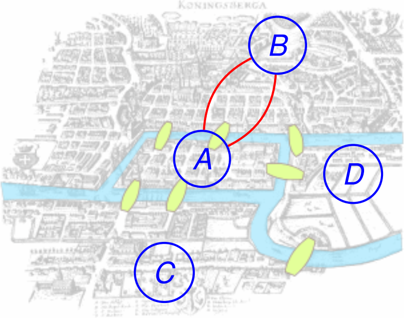

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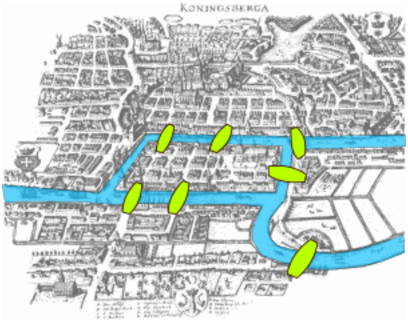

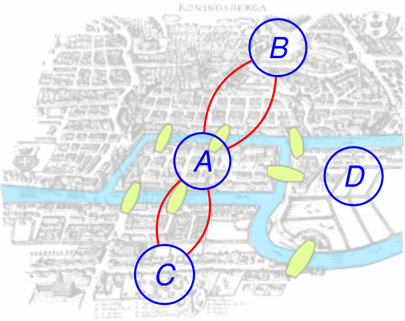

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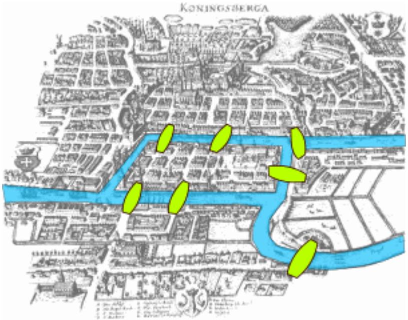

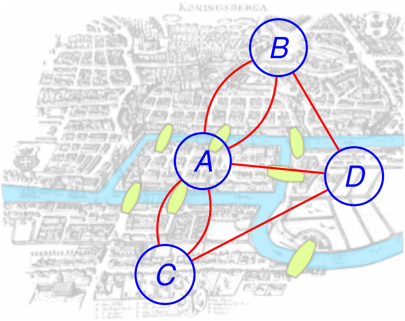

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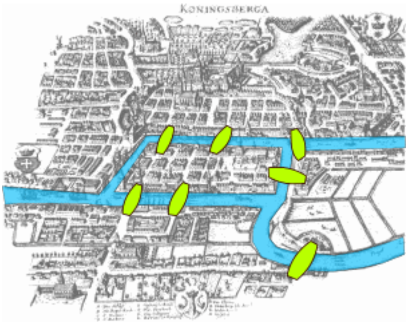

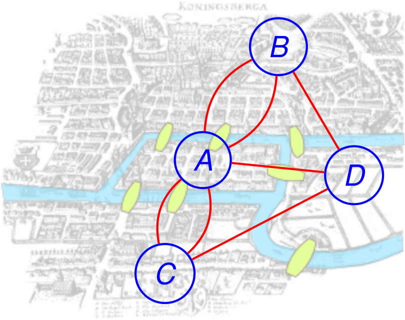

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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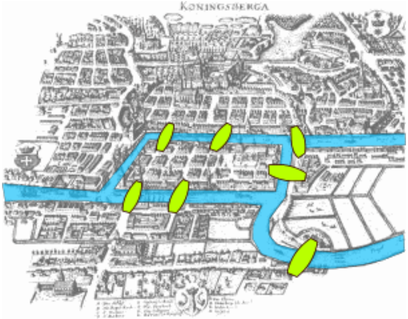

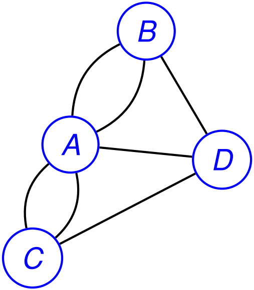

Can you draw a tour in the graph where you visit each edge once?   Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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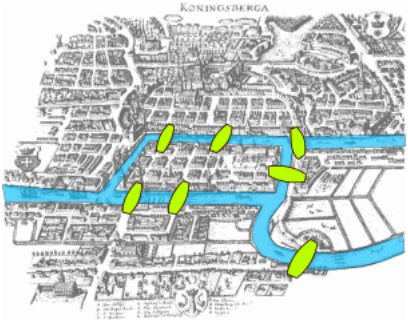

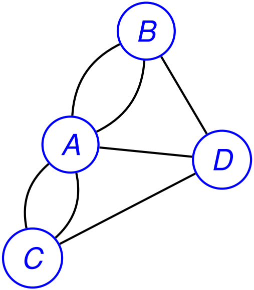

Can you draw a tour in the graph where you visit each edge once?   Yes?   No?

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Can you make a tour visiting each bridge exactly once?

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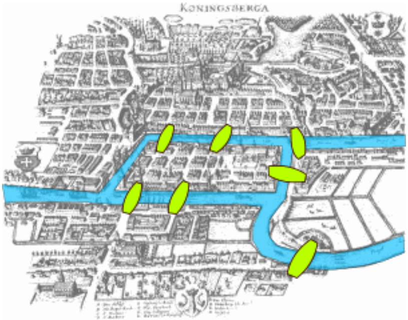
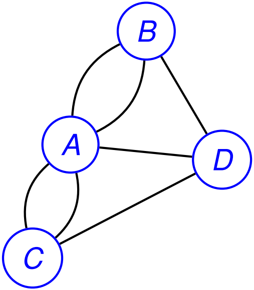
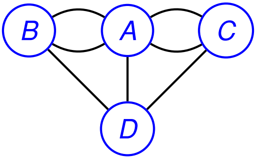




Can you draw a tour in the graph where you visit each edge once? Yes? No?

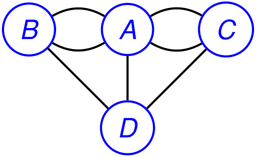
We will see!

Graphs: formally.



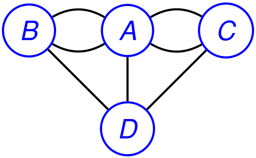
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

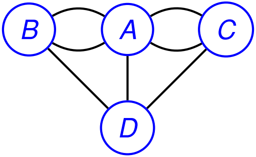
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

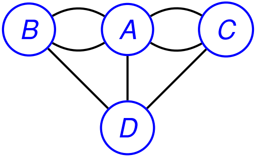


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



Graph: $G = (V, E)$.

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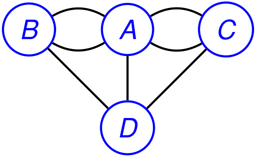
$E \subseteq V \times V$ -

Graphs: formally.



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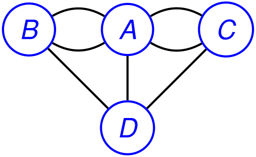


Graph: $G = (V, E)$.

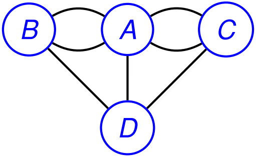
V - set of vertices.

$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



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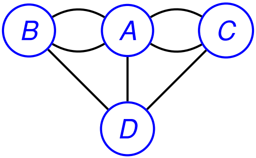

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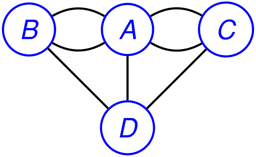

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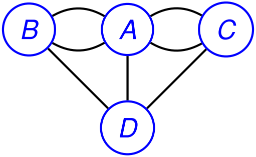
Graphs: formally.



Graph: $G = (V,E)$.

V - set of vertices.

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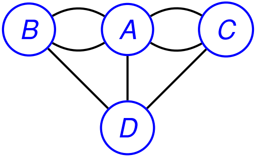
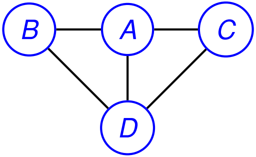
$E \subseteq V \times V$ - set of edges.

$\{\{A,B\},\{A,B\},\{A,C\},\{A,C\},\{B,D\},\{A,D\},\{C,D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

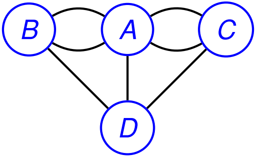
$E \subseteq V \times V$ - set of edges.

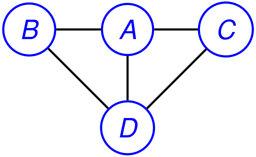
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

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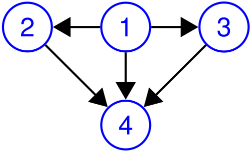
For CS 70, usually simple graphs.

No parallel edges.

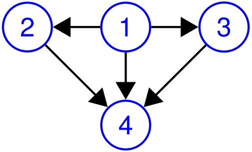
Multigraph above.

Directed Graphs



$G = (V, E)$.

Directed Graphs



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V - set of vertices.

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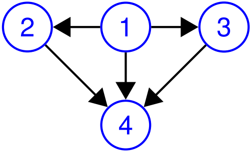
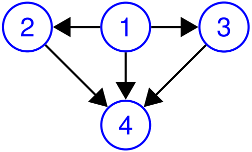


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



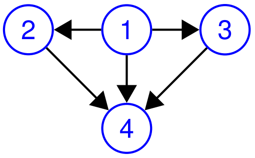
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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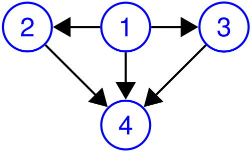
E ordered pairs of vertices.

$\{(1, 2),$

Directed Graphs



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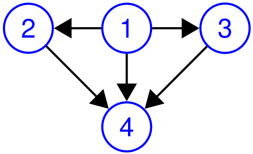
V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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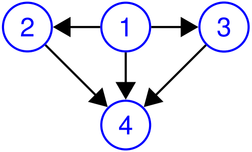
Directed Graphs



$G = (V, E)$.

V - set of vertices.

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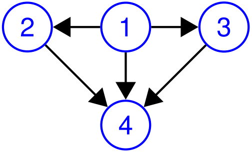
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

Directed Graphs



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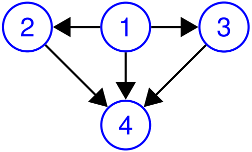
One way streets.

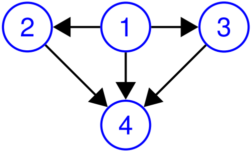
Directed Graphs



$G = (V, E)$.
V - set of vertices.
$\{1, 2, 3, 4\}$
E ordered pairs of vertices.
$\{(1,2), (1,3), (1,4), (2,4), (3,4)\}$

One way streets.
Tournament:

Directed Graphs



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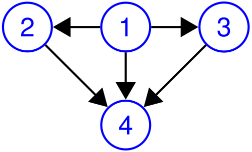
One way streets.

Tournament: 1 beats 2,

Directed Graphs



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V - set of vertices.
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E ordered pairs of vertices.
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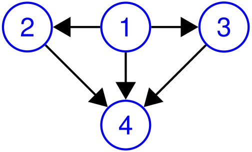
One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



$G = (V, E)$.

V - set of vertices.

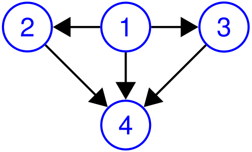
$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



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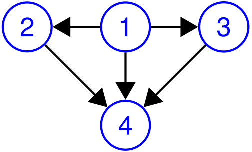
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

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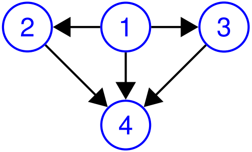
Social Network:

Directed Graphs



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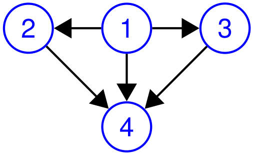
Directed Graphs



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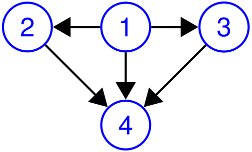
Directed Graphs



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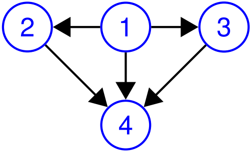
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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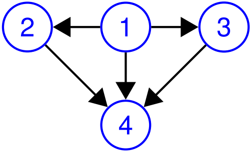

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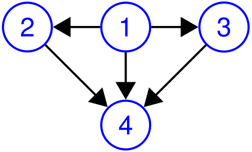
Likes.

Directed Graphs



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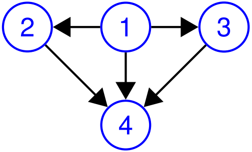
Directed Graphs



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Graph Concepts and Definitions.

Graph: $G = (V, E)$

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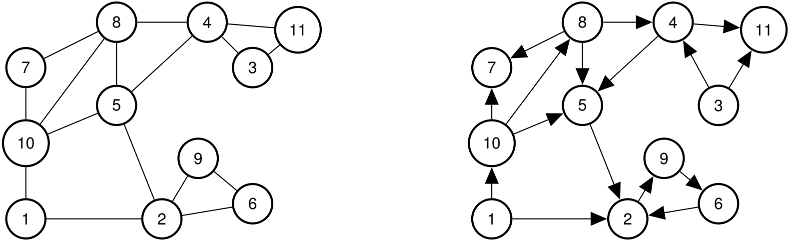

Neighbors of 10?

Graph Concepts and Definitions.

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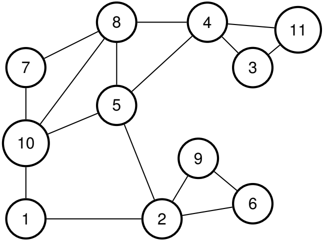

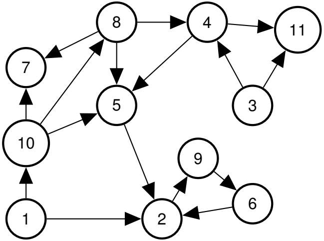

Neighbors of 10? 1,

Graph Concepts and Definitions.

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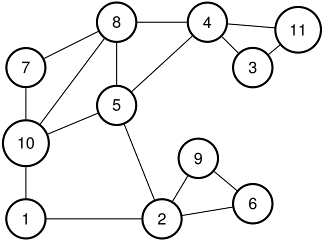

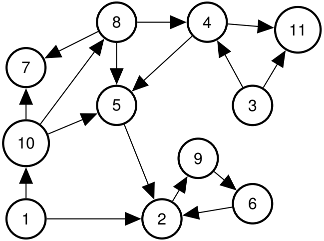

Neighbors of 10? 1,5,

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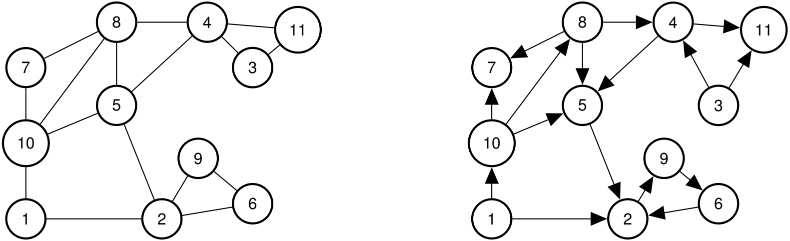

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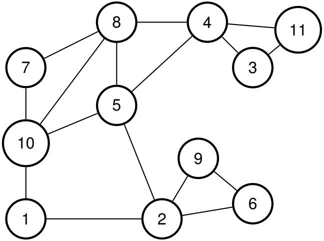

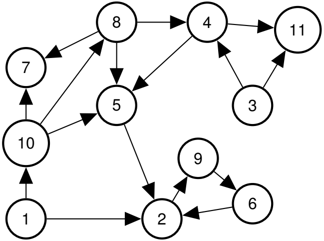

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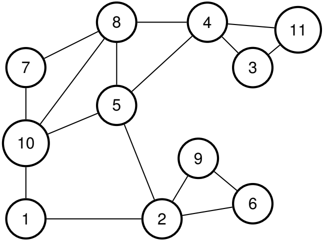

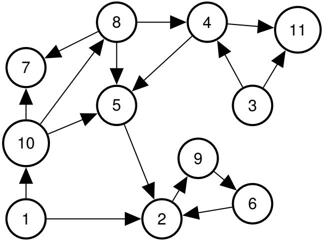

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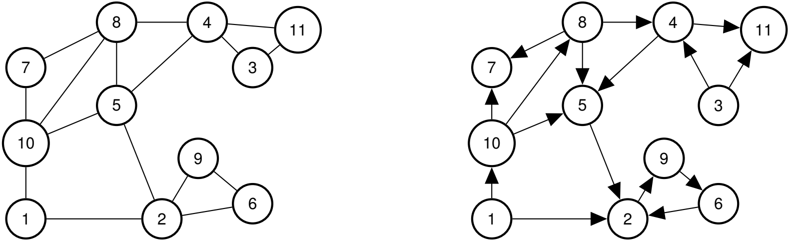

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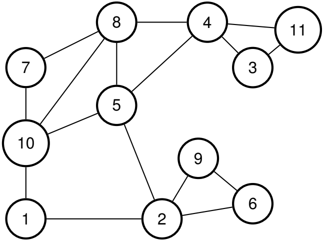

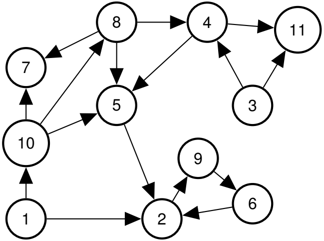

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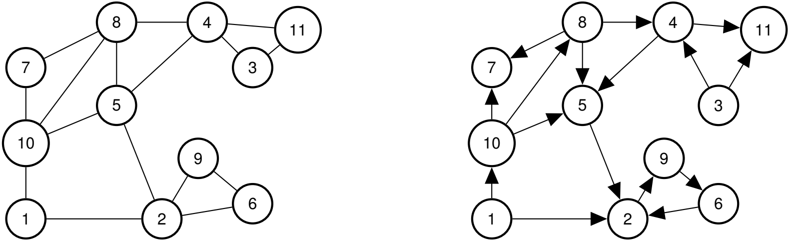

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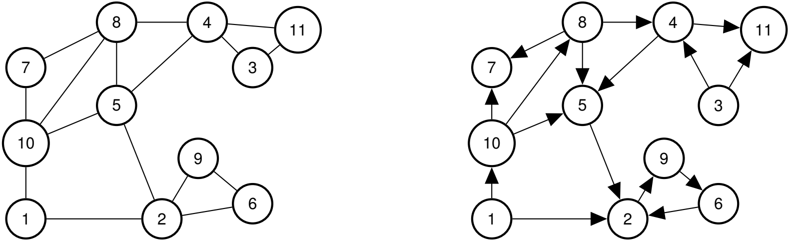

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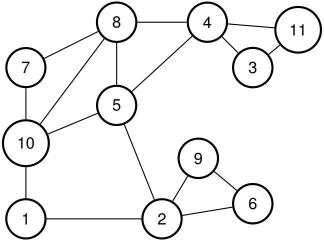

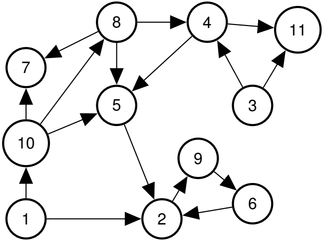

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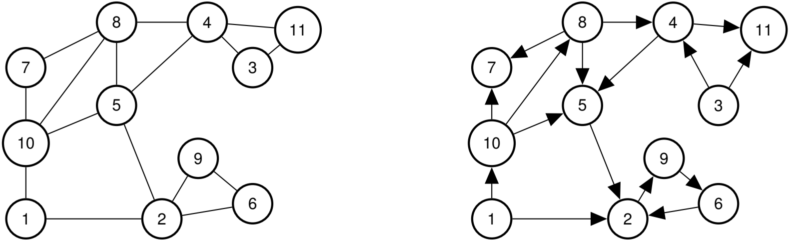

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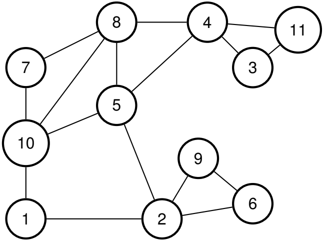

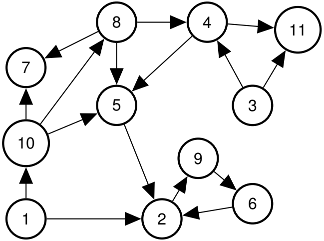

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Directed graph?

Graph Concepts and Definitions.

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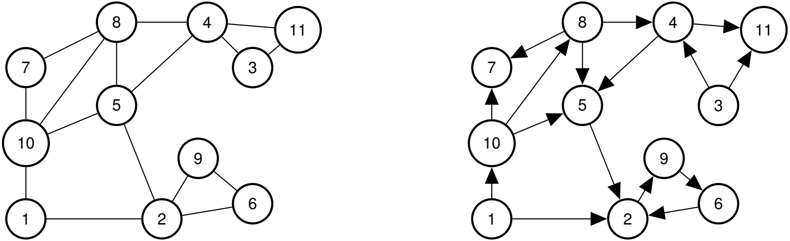

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Directed graph?

In-degree of 10?

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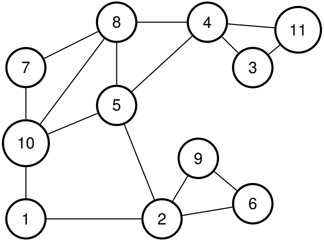

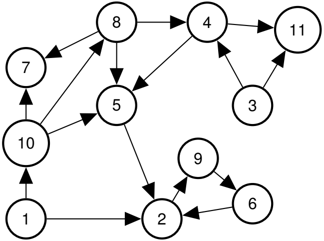

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Directed graph?

In-degree of 10? 1

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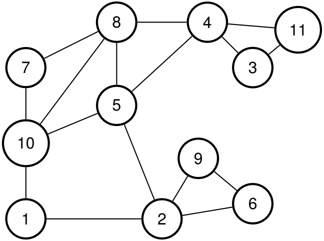

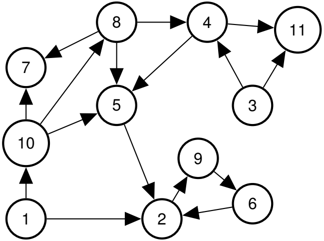

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In-degree of 10? 1   Out-degree of 10?

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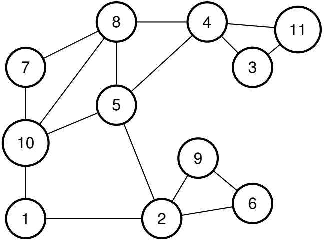

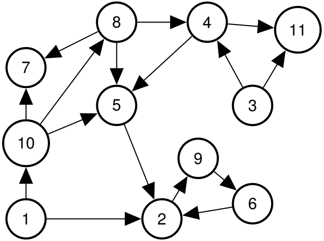

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Directed graph?

In-degree of 10? 1   Out-degree of 10? 3

Graph Concepts and Definitions.

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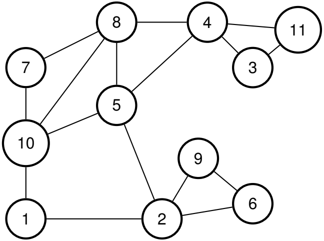
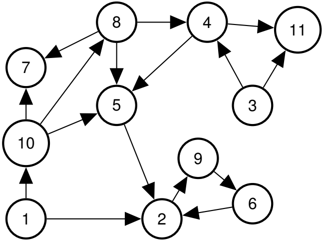
neighbors, adjacent, degree, incident, in-degree, out-degree





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Quick Proof.

The sum of the vertex degrees is equal to

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(A) the total number of vertices, $|V|$.

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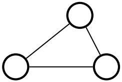

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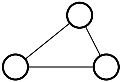

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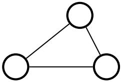

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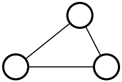

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What could it be?

Quick Proof.

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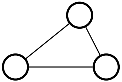


Not (B)! Triangle.

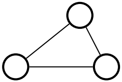
What could it be? For triangle number of edges is 3, the sum of degrees is 6.

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) Something else?

Not (A)! Triangle.

 Not (B)! Triangle.

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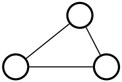

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What could it be? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

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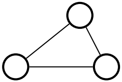

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How many incidences does each edge contribute?

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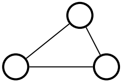

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How many incidences does each edge contribute? 2.

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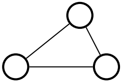

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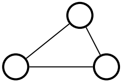

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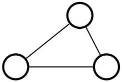

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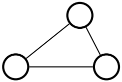

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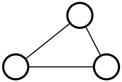

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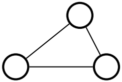

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Thm: Sum of vertex degrees is $2|E|$.

Proof of “handshake” lemma.

Lemma: The sum of degrees is $2|E|$, for a graph $G = (V, E)$.

The number of edge-vertex incidences for an edge e is 2.

The total number of edge-vertex incidences is $2|E|$.

The sum of degrees is $2|E|$.

Proof of "handshake" lemma.

Lemma: The sum of degrees is $2|E|$, for a graph $G = (V, E)$.

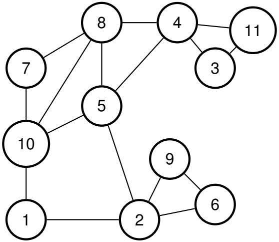
The number of edge-vertex incidences for an edge e is 2.

The total number of edge-vertex incidences is $2|E|$.

The sum of degrees is $2|E|$.

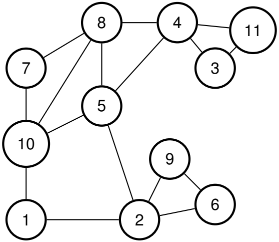
Handshake lemma: sum of number of handshakes of each person is twice the number of handshakes.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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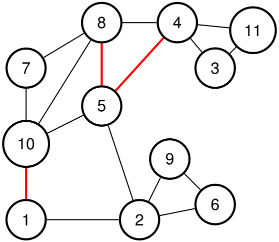

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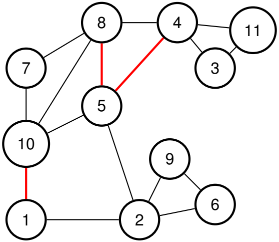
Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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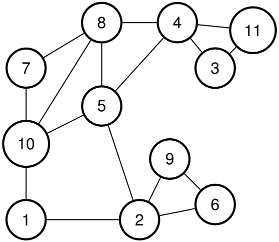

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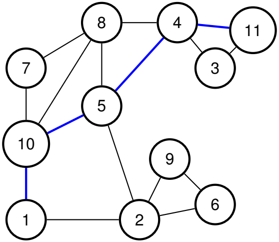

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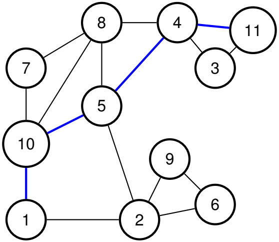
Paths, walks, cycles, tour.



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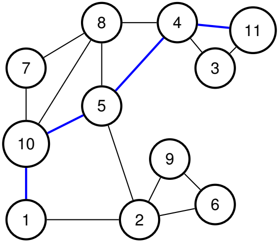

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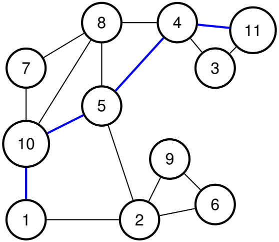
Paths, walks, cycles, tour.



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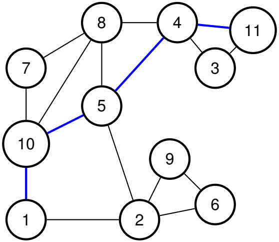
Paths, walks, cycles, tour.



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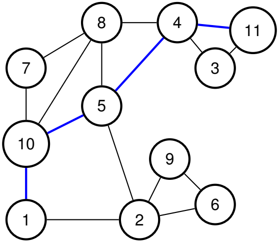
Quick Check! Length of path?

Paths, walks, cycles, tour.



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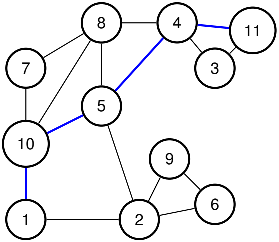
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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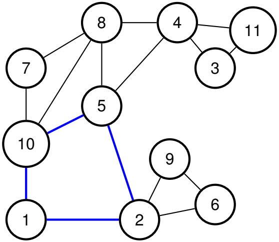
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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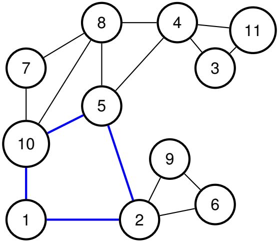
Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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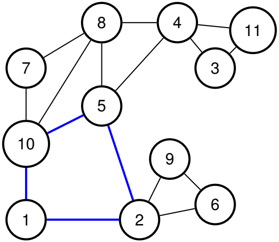
Paths, walks, cycles, tour.



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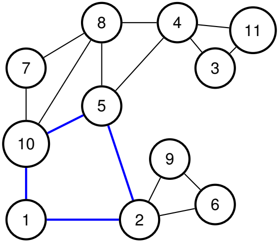
Paths, walks, cycles, tour.



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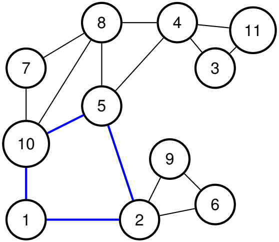
Paths, walks, cycles, tour.



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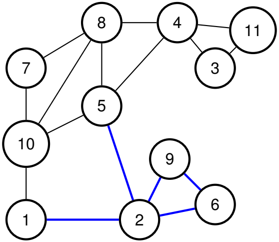
Paths, walks, cycles, tour.



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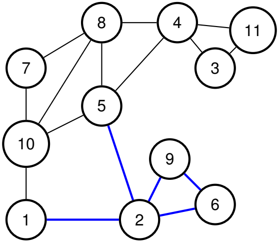
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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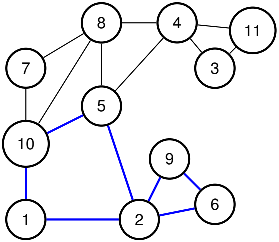

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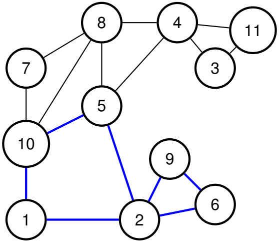
Paths, walks, cycles, tour.



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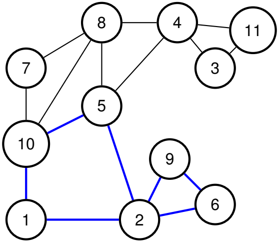

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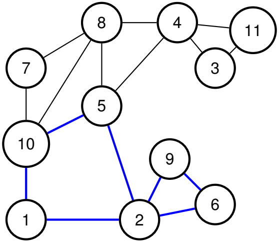

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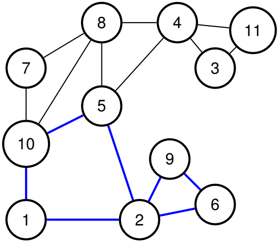
Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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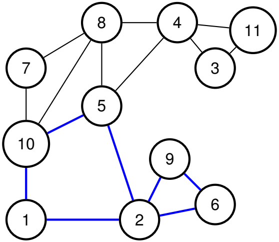
Path is to Walk as Cycle is to ?? Tour!

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

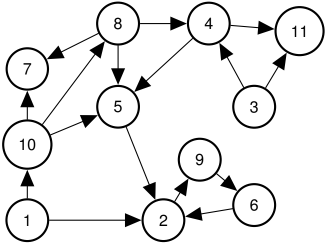
Path: $(v_1, v_2), (v_2, v_3), \ldots (v_{k-1}, v_k)$.
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually *simple*. No repeated vertex!
Walk is sequence of edges with possible repeated vertex or edge.

Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ?? Tour!

Directed Paths.



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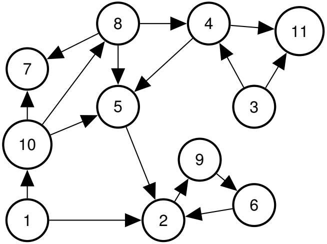

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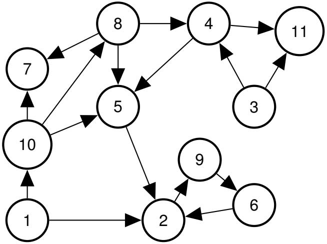

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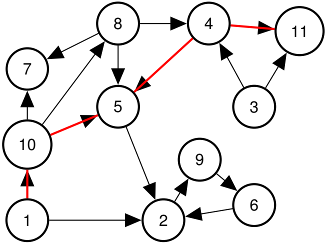

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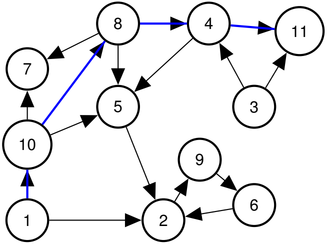

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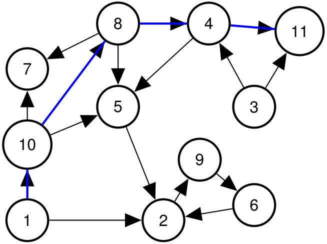

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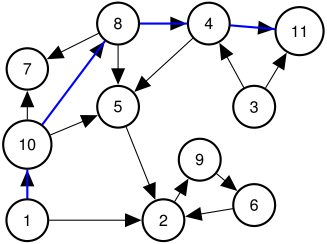

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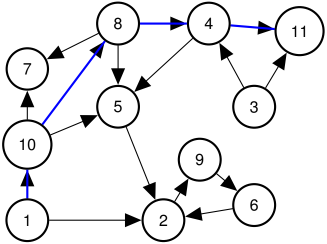

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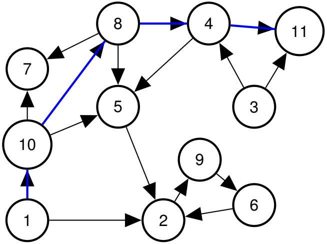

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Paths, walks, cycles, tours

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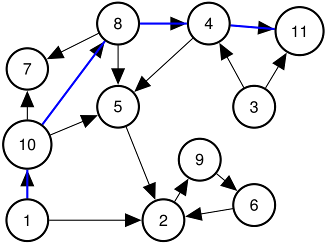

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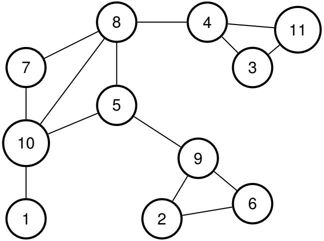
Paths, walks, cycles, tours ... are analogous to undirected now.

Connectivity



u and *v* are connected if there is a path between *u* and *v*.

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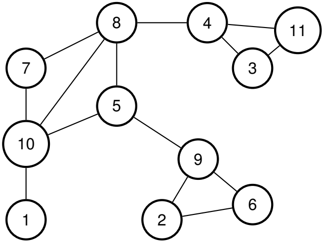

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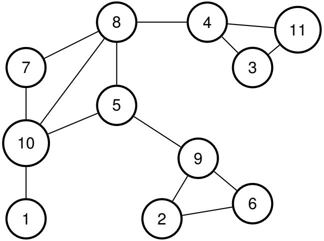

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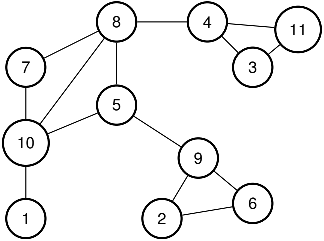

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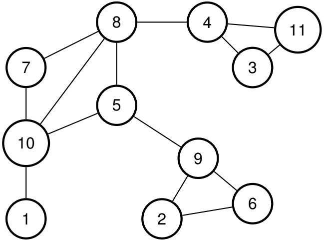

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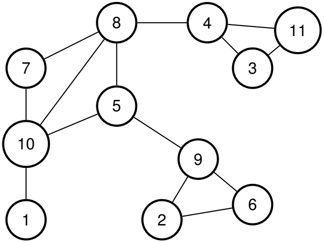

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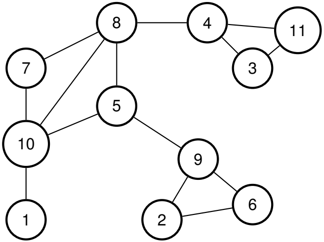

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Proof:

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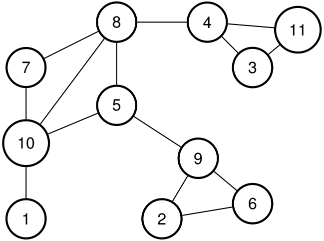

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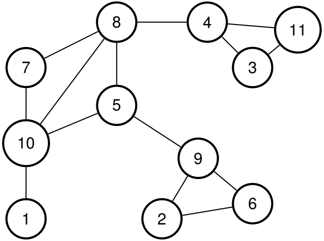

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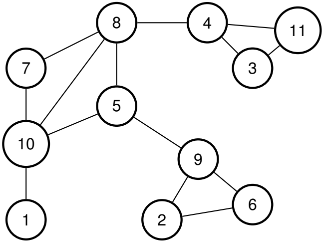

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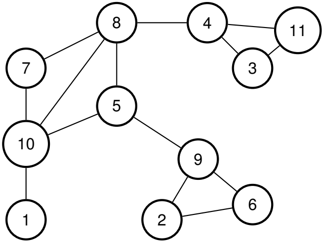

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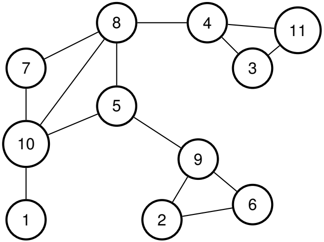

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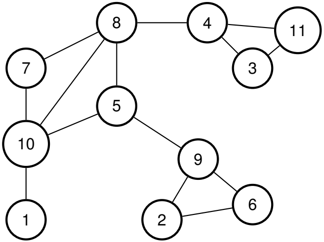

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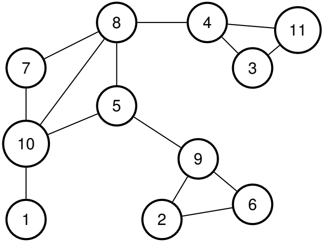
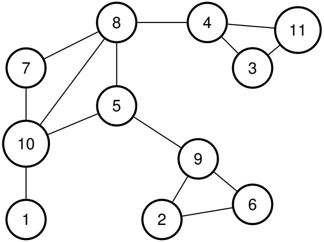


Is graph above connected?



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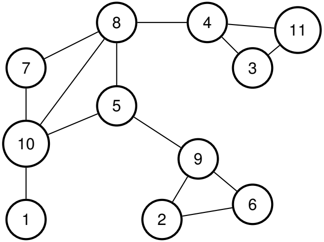
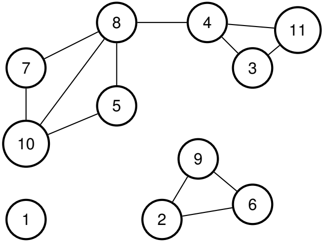


Is graph above connected? Yes!

How about now?



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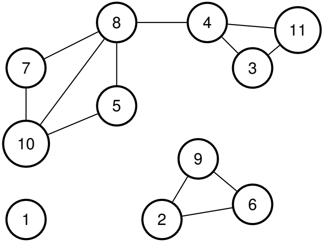

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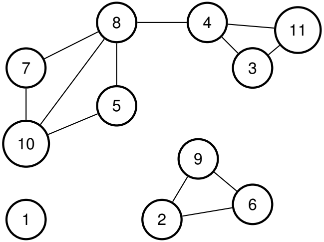
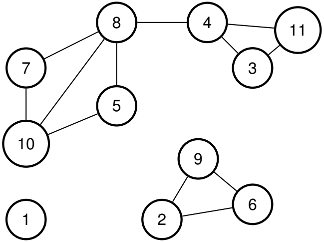
Connected Components?



Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.
A connected component is a maximal set of connected nodes in a graph.
Quick Check: Is $\{10, 7, 5\}$ a connected component?



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Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.

Proof of only if: Eulerian $\implies$ connected and all even degree.

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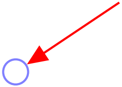

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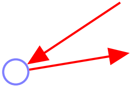

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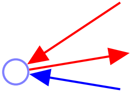

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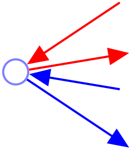

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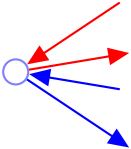

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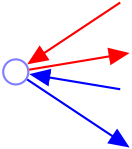

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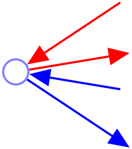

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For starting node, tour leaves first

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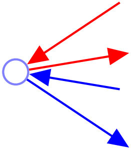

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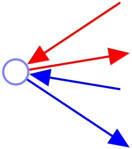

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Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

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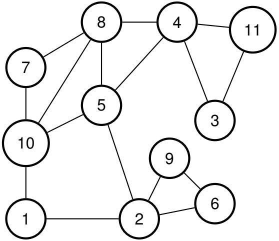

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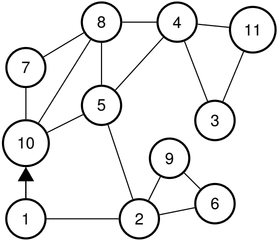

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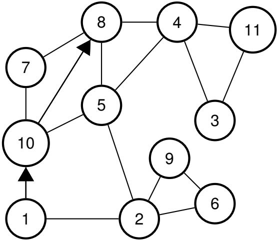

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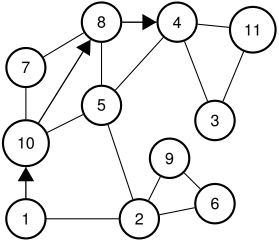

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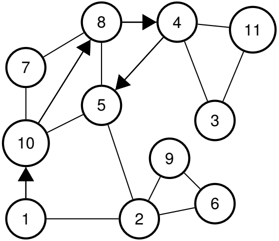

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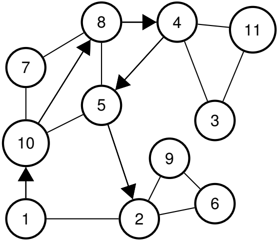

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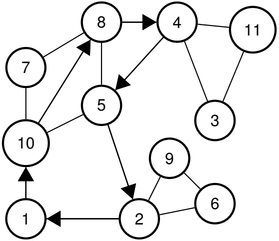

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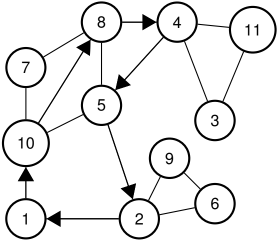

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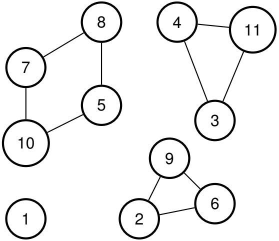

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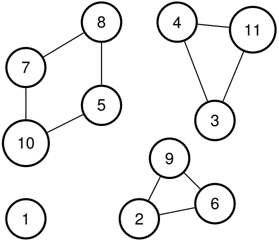

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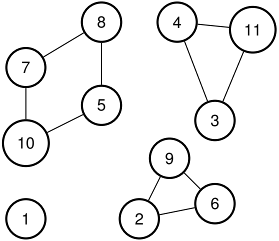

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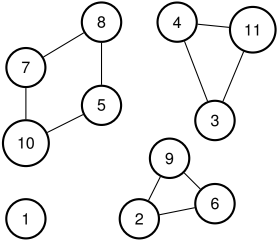

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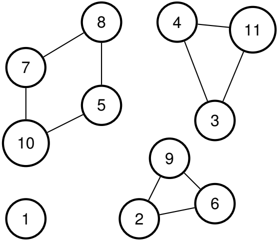

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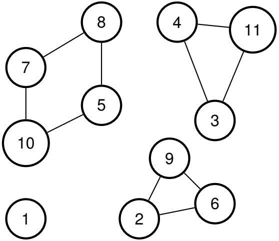

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   Example: $v_1 = 1$,

Finding a tour!

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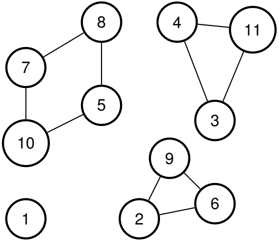

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   Example: $v_1 = 1$, $v_2 = 10$,

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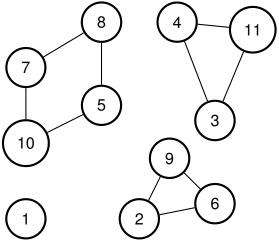

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   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

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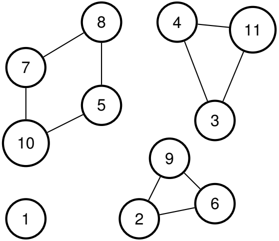

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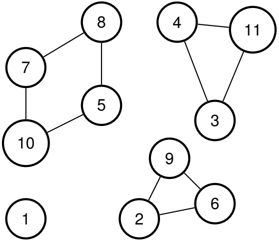

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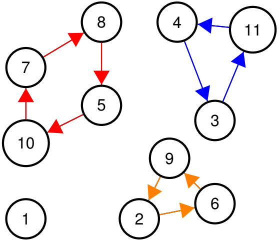

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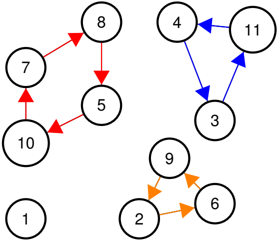

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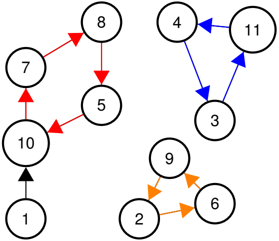

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   1,10

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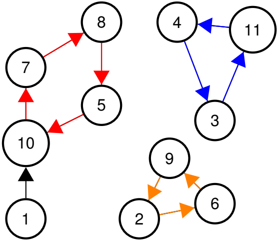

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   1,10,7,8,5,10

Finding a tour!

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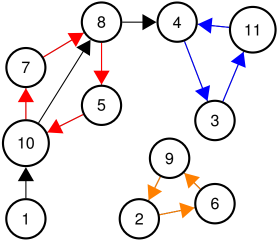

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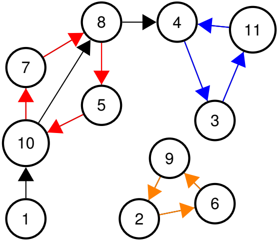

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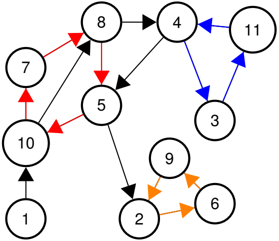

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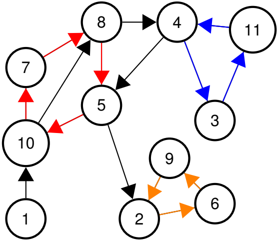

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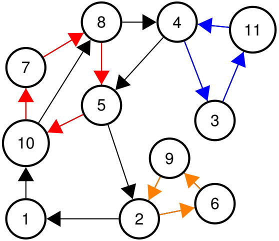

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Claim: Do get back to v!
Proof of Claim: Even degree.

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Claim: Do get back to v!

Proof of Claim: Even degree. If enter, can leave

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!

Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
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2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

General case: Recursive algorithm, proof by induction.

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

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Proof of Claim: Even degree. If enter, can leave except for v. □

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

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Proof of Claim: Even degree. If enter, can leave except for v. □

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
Prf: Tour C has even incidences to any vertex v.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

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3. Find tour T_i of G_i

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3. Find tour T_i of G_i starting/ending at v_i.

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4. Splice T_i into C where v_i first appears in C.

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Visits every edge once:
Visits edges in C

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Visits every edge once:
Visits edges in C exactly once.

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Visits every edge once:
Visits edges in C exactly once.
By induction for all other edges by induction on G_i.

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