1 Perfect Square

Note 2

(a) Prove that if n^2 is odd, then n must also be odd.

(b) Prove that if n^2 is odd, then n^2 can be written in the form $8k+1$ for some integer k.

2 Numbers of Friends

Note 2

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.

(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

3 Pebbles

Note 2

Suppose you have a rectangular array of pebbles, where each pebble is either red or blue. Suppose that for every way of choosing one pebble from each column, there exists a red pebble among the chosen ones.

Prove that there must exist an all-red column.