

1 Probability Potpourri

Note 13
Note 14

Provide brief justification for each part.

- (a) For two events A and B in any probability space, show that $\mathbb{P}[A \setminus B] \geq \mathbb{P}[A] - \mathbb{P}[B]$.
- (b) Suppose $\mathbb{P}[D | C] = \mathbb{P}[D | \bar{C}]$, where \bar{C} is the complement of C . Prove that D is independent of C .
- (c) If A and B are disjoint, does that imply they're independent?

3 Balls and Bins

Note 14

Suppose you throw n balls into n labeled bins one at a time.

(a) What is the probability that the first bin is empty?

(b) What is the probability that the first k bins are empty?

(c) Let A be the event that at least k bins are empty. Let m be the number of subsets of k bins out of the total n bins. If we assume A_i is the event that the i th set of k bins is empty. Then we can write A as the union of A_i 's:

$$A = \bigcup_{i=1}^m A_i.$$

Compute m in terms of n and k , and use the union bound to give an upper bound on the probability $\mathbb{P}[A]$.

(d) What is the probability that the second bin is empty given that the first one is empty?

(e) Are the events that “the first bin is empty” and “the first two bins are empty” independent?

(f) Are the events that “the first bin is empty” and “the second bin is empty” independent?