Counting and Probability

Second half of the semester: Probability.

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A bag contains a set of colored balls:

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What is the chance that a ball taken from the bag is blue?

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Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?

Count blue.

Counting and Probability

Second half of the semester: Probability.

A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Counting and Probability

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What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

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After the Midterm: Probability.

Counting and Probability

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A bag contains a set of colored balls:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

After the Midterm: Probability. Professor Sinclair.

Outline

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Count?

How many outcomes possible for k coin tosses?
How many handshakes for n people?
How many 10 digit numbers?
How many 10 digit numbers without repeating digits?

Using a tree of possibilities...

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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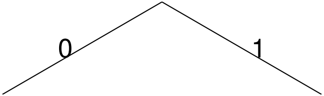

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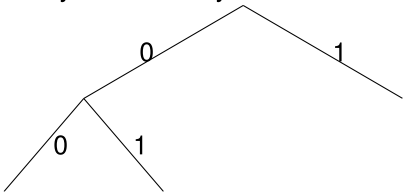

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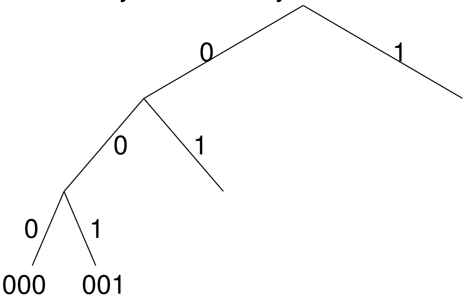

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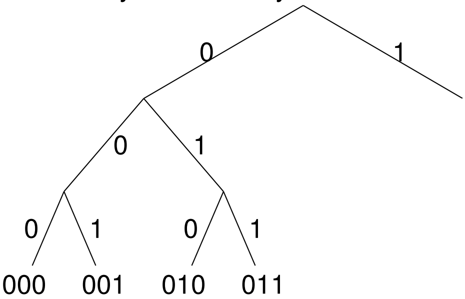

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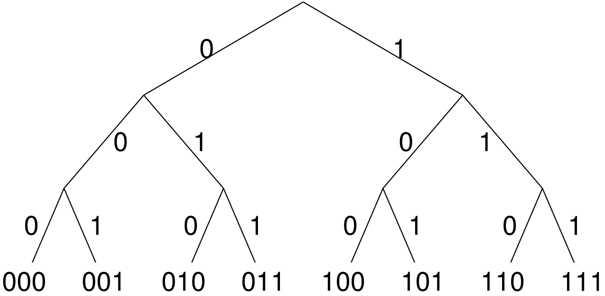

8 leaves which is $2 \times 2 \times 2$.

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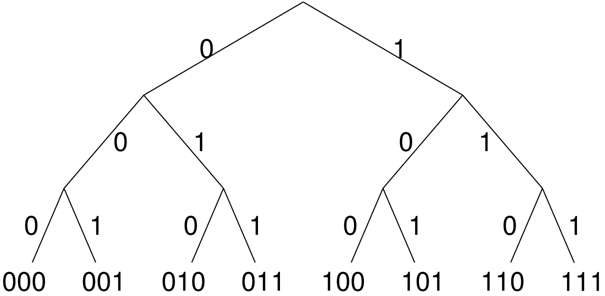

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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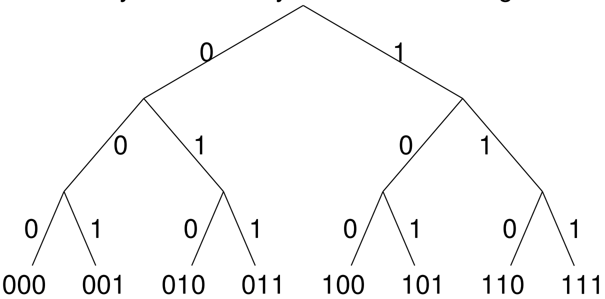

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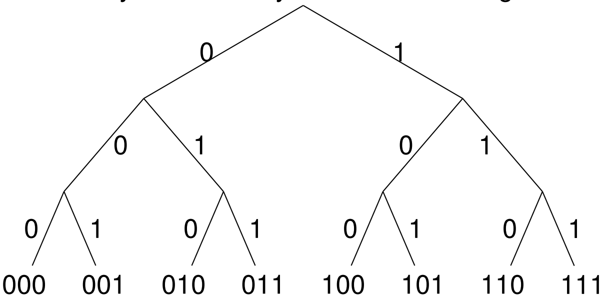

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8 3-bit strings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, …, then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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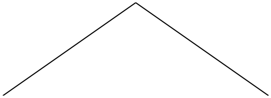

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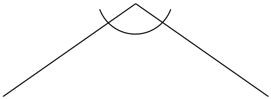

n_1

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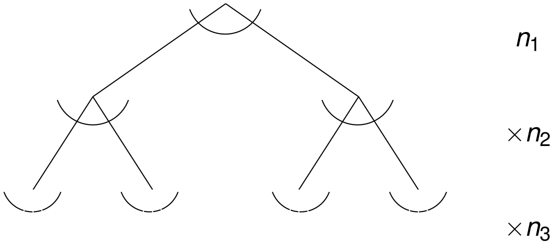

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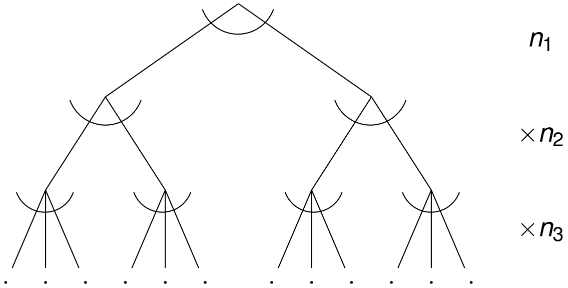

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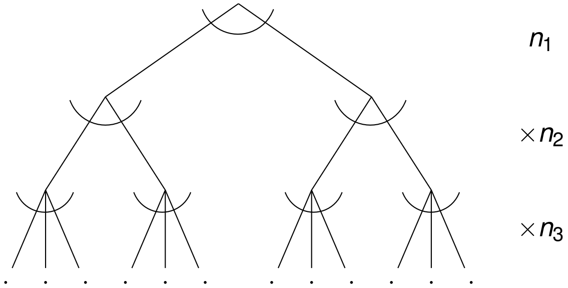

In picture, $2 \times 2 \times 3 = 12$

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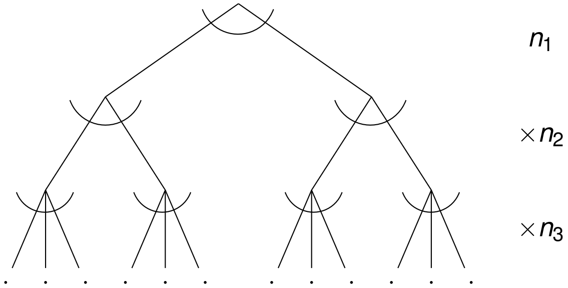

In picture, $2 \times 2 \times 3 = 12$

Using the first rule..

How many outcomes possible for k coin tosses?

Using the first rule..

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2 ways for first choice,

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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How many 10 digit numbers (leading zeroes are OK)?

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9 ways for first choice, 10 ways for second choice, ...
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How many functions f mapping S to T?

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Permutations.

[1] By definition: $0! = 1$. $n! = n(n-1)(n-2)\ldots 1$.

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How many orderings of n objects are there?
Permutations of n objects.

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Permutations.

How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third, ...

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How many different samples of size k from n numbers **without replacement.**

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... $n * (n-1) * (n-2) \cdot * (n-k+1) = \frac{n!}{(n-k)!}$.

How many orderings of n objects are there?
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One-to-One Functions.

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$

A one-to-one function (from S to S) is a permutation!

Counting sets..when order doesn't matter.

How many sets of 5 playing cards ("poker hands")?

[2]When each unordered object corresponds equal numbers of ordered objects.

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Are $A, K, Q, 10, J$ of spades
and $10, J, Q, K, A$ of spades the same?

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Second Rule of Counting: If order doesn't matter count ordered objects and then divide by number of orderings.[2]

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Number of orderings for a poker hand: 5!

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$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

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$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2] When each unordered object corresponds equal numbers of ordered objects.

When order doesn't matter.

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Choose 2 out of n?

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$$\frac{n \times (n-1)}{}$$

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Notation: $\binom{n}{k}$ **and pronounced "n choose k."**

Simple Practice.

How many orderings of letters of CAT?

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3 ways to choose first letter, 2 ways to choose second, 1 for last.
$\implies 3 \times 2 \times 1 = 3!$ orderings

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How many orderings of the letters in ANAGRAM?

Ordered, except for A!
total orderings of 7 letters. 7!
total "extra counts" or orderings of two A's? 3!
Total orderings? $\frac{7!}{3!}$

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How many orderings of letters in MISSISSIPPI?

Simple Practice.

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Ordered, except for A!
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Total orderings? $\frac{7!}{3!}$

How many orderings of letters in MISSISSIPPI?

4 S's, 4 I's, 2 P's.
11 letters total!
11! ordered objects!
$4! \times 4! \times 2!$ ordered objects per "unordered object"
$\implies \frac{11!}{4!4!2!}$.

Sampling...

Sample k items out of n

Sampling...

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Without replacement:

Sampling...

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Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n - 1 \times n - 2 \ldots \times n - k + 1$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "$k!$"

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"n choose k"

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Sampling...

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"n choose k"

With Replacement.

Order matters: $n \times n$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

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“n choose k”

With Replacement.

Order matters: $n \times n \times \ldots n$

Sampling...

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Order matters: $n \times n \times \ldots n = n^k$

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Problem: depends on how many of each item we chose!

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Set: $1, 2, 3$

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Set: $1,2,3$ 3! orderings map to it.

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Sample k items out of n

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Set: $1,2,2$ $\frac{3!}{2!}$ orderings map to it.

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Problem: depends on how many of each item we chose!

Set: $1,2,3$ — $3!$ orderings map to it.
Set: $1,2,2$ — $\frac{3!}{2!}$ orderings map to it.

How do we deal with this situation?!?!

New Technique: Stars and Bars....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), see what results.

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5 dollars for Bob and 0 for Alice:

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5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

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5 ordered sets: (A,B,B,B,B) ; (B,A,B,B,B); ...

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4 for Bob and 1 for Alice:
5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

Well, we can list the possibilities.
$0+5$, $1+4$, $2+3$, $3+2$, $4+1$, $5+0$.

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For 2 numbers adding to k, we get $k+1$.

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$0+5$, $1+4$, $2+3$, $3+2$, $4+1$, $5+0$.

For 2 numbers adding to k, we get $k+1$.

For 3 numbers adding to k? More than 3?

Stars and Bars.

How many ways to add up n natural numbers to equal k?

Stars and Bars.

How many ways to add up n natural numbers to equal k?

Or: k choices from set of n possibilities with replacement.
Sample with replacement where order just doesn't matter.

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How many ways can Alice, Bob, and Eve split 5 dollars.

Stars and Bars.

How many ways to add up n natural numbers to equal k?

Or: k choices from set of n possibilities with replacement.
Sample with replacement where order just doesn't matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: $\star\star\star\star\star$.

Stars and Bars.

How many ways to add up n natural numbers to equal k?

Or: k choices from set of n possibilities with replacement.
Sample with replacement where order just doesn't matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: $\star\star\star\star\star$.

Alice: 2, Bob: 1, Eve: 2.

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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

Stars and Bars.

How many different 5 star and 2 bar diagrams?

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$$\binom{n+k-1}{n-1}$$

Stars and Bars Poll

Mark what's correct:

(A) ways to split 5 dollars among 3: $\binom{7}{2}$

(B) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(C) ways to split 3 dollars among 5: $\binom{7}{5}$

(D) ways to split 5 dollars among 3: $\binom{7}{5}$

Stars and Bars Poll

Mark what's correct:

(A) ways to split 5 dollars among 3: $\binom{7}{2}$

(B) ways to split n dollars among k: $\binom{n+k-1}{k-1}$

(C) ways to split 3 dollars among 5: $\binom{7}{5}$

(D) ways to split 5 dollars among 3: $\binom{7}{5}$

(A),(B),(D) are correct.

Combinatorial Proofs - 1

A technique to prove identities by counting arguments!

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Choose a subset of size $n-k$

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and what's left out

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Choosing a subset of size k is same

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Binomial Theorem

Theorem: $2^n = \binom{n}{n} + \binom{n}{n-1} + \cdots + \binom{n}{0}$

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Sum over i to get total number of subsets..which is also 2^n. $\square$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
Used to reason about all subsets
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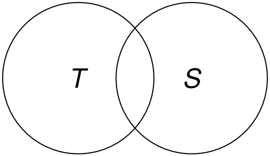

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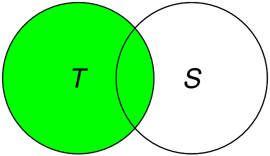

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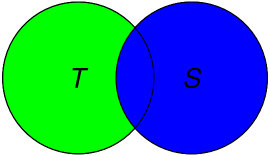

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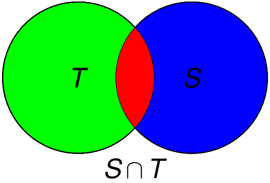

In T. $\implies$ $|T|$
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Elements in $S \cap T$ are counted twice.

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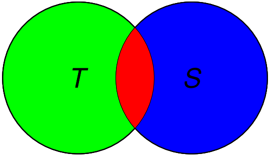
For any S and T, $|S \cup T| = |S| + |T| - |S \cap T|$.



$S \cap T$

In T. $\implies |T|$
In S. $\implies + \ |S|$
Elements in $S \cap T$ are counted twice.
Subtract. $\implies -|S \cap T|$

Simple Inclusion/Exclusion

Sum Rule: For disjoint sets S and T, $|S \cup T| = |S| + |T|$
Used to reason about all subsets
by adding number of subsets of size 1, 2, 3,...

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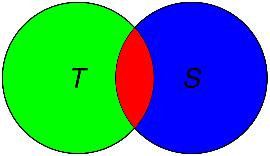

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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....and more to come! Probability Theory!

Wrapup.

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Good Studying and Good Luck!!!