Lecture 7 Outline.

1. Modular Arithmetic.

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   Clock Math!!!

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1. Modular Arithmetic.
   Clock Math!!!
2. Inverses for Modular Arithmetic: Greatest Common Divisor (GCD).
3. Euclid's GCD Algorithm

Clock Math

If it is 4:00 now.

Clock Math

If it is 4:00 now.
What time is it in 5 hours?

Clock Math

If it is 4:00 now.
What time is it in 5 hours? 9:00!

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours?

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Clock Math

If it is 4:00 now.
- What time is it in 5 hours? 9:00!
- What time is it in 15 hours? 19:00!
  - Actually 7:00.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the "same as 7" with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the "same as 7" with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours?

Clock Math

If it is 4:00 now.

- What time is it in 5 hours? 9:00!
- What time is it in 15 hours? 19:00!
  - Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00!

Clock Math

If it is 4:00 now.

- What time is it in 5 hours? 9:00!
- What time is it in 15 hours? 19:00!
  - Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00! or 8:00.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00! or 8:00.

8 is the same as 104 for a 12 hour clock system.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

What time is it in 15 hours? 19:00!

Actually 7:00.

19 is the “same as 7” with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00! or 8:00.

8 is the same as 104 for a 12 hour clock system.

Clock time equivalent up to addition of any integer multiple of 12.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

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Actually 7:00.

19 is the "same as 7" with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00! or 8:00.

8 is the same as 104 for a 12 hour clock system.

Clock time equivalent up to addition of any integer multiple of 12.

Clock Math

If it is 4:00 now.

What time is it in 5 hours? 9:00!

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Actually 7:00.

19 is the "same as 7" with respect to a 12 hour clock system.

Clock time equivalent up to to addition/subtraction of 12.

What time is it in 100 hours? 104:00! or 8:00.

8 is the same as 104 for a 12 hour clock system.

Clock time equivalent up to addition of any integer multiple of 12.

Custom is only to use the representative in $\{1, \ldots, 11, 12\}$

Day of the week.

Today is Tuesday.

Day of the week.

Today is Tuesday.
What day is it a year from now?

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now. day 6

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now. day 6 or Saturday.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now. day 6 or Saturday.

24 days from now.

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
4 days from now. day 6 or Saturday.
24 days from now. day 26

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now. day 6 or Saturday.

24 days from now. day 26 or day 5, which is Friday!

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
4 days from now. day 6 or Saturday.
24 days from now. day 26 or day 5, which is Friday!
two days are equivalent up to addition/subtraction of multiple of 7.

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.

4 days from now. day 6 or Saturday.

24 days from now. day 26 or day 5, which is Friday!

two days are equivalent up to addition/subtraction of multiple of 7.

10 days from now

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
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Today: day 2.
4 days from now. day 6 or Saturday.
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two days are equivalent up to addition/subtraction of multiple of 7.
10 days from now is day 5 again, Friday!

Day of the week.

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What day is it a year from now? on February 6, 2025?
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0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

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4 days from now. day 6 or Saturday.
24 days from now. day 26 or day 5, which is Friday!
two days are equivalent up to addition/subtraction of multiple of 7.
10 days from now is day 5 again, Friday!

What day is it a year from now?

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
4 days from now. day 6 or Saturday.
24 days from now. day 26 or day 5, which is Friday!
two days are equivalent up to addition/subtraction of multiple of 7.
10 days from now is day 5 again, Friday!

What day is it a year from now?
This year is a leap year!

Day of the week.

Today is Tuesday.

What day is it a year from now? on February 6, 2025?

Number days.

0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.

4 days from now. day 6 or Saturday.

24 days from now. day 26 or day 5, which is Friday!

two days are equivalent up to addition/subtraction of multiple of 7.

10 days from now is day 5 again, Friday!

What day is it a year from now?

This year is a leap year! So 366 days from now.

Day of the week.

Today is Tuesday.
  What day is it a year from now? on February 6, 2025?
    Number days.
      0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
  4 days from now. day 6 or Saturday.
  24 days from now. day 26 or day 5, which is Friday!
    two days are equivalent up to addition/subtraction of multiple of 7.
    10 days from now is day 5 again, Friday!

What day is it a year from now?
  This year is a leap year! So 366 days from now.
  Day 2+366 or day 368.

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
4 days from now. day 6 or Saturday.
24 days from now. day 26 or day 5, which is Friday!
two days are equivalent up to addition/subtraction of multiple of 7.
10 days from now is day 5 again, Friday!

What day is it a year from now?
This year is a leap year! So 366 days from now.
Day 2+366 or day 368.
Smallest representation:

Day of the week.

Today is Tuesday.
What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
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24 days from now. day 26 or day 5, which is Friday!
two days are equivalent up to addition/subtraction of multiple of 7.
10 days from now is day 5 again, Friday!

What day is it a year from now?
This year is a leap year! So 366 days from now.
Day 2+366 or day 368.
Smallest representation:
subtract 7 until smaller than 7.

Day of the week.

Today is Tuesday.
  What day is it a year from now? on February 6, 2025?
    Number days.
      0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
  4 days from now. day 6 or Saturday.
  24 days from now. day 26 or day 5, which is Friday!
    two days are equivalent up to addition/subtraction of multiple of 7.
    10 days from now is day 5 again, Friday!

What day is it a year from now?
  This year is a leap year! So 366 days from now.
  Day 2+366 or day 368.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.

Day of the week.

Today is Tuesday.
  What day is it a year from now? on February 6, 2025?
    Number days.
      0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
  4 days from now. day 6 or Saturday.
  24 days from now. day 26 or day 5, which is Friday!
    two days are equivalent up to addition/subtraction of multiple of 7.
    10 days from now is day 5 again, Friday!

What day is it a year from now?
  This year is a leap year! So 366 days from now.
  Day 2+366 or day 368.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.
  368/7

Day of the week.

Today is Tuesday.
  What day is it a year from now? on February 6, 2025?
    Number days.
      0 for Sunday, 1 for Monday, ..., 6 for Saturday.

Today: day 2.
  4 days from now. day 6 or Saturday.
  24 days from now. day 26 or day 5, which is Friday!
    two days are equivalent up to addition/subtraction of multiple of 7.
    10 days from now is day 5 again, Friday!

What day is it a year from now?
  This year is a leap year! So 366 days from now.
  Day 2+366 or day 368.
Smallest representation:
  subtract 7 until smaller than 7.
  divide and get remainder.
  368/7 leaves quotient of 52 and remainder 4.

Day of the week.

Today is Tuesday.
 What day is it a year from now? on February 6, 2025?
  Number days.
   0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

Today: day 2.
 4 days from now. day 6 or Saturday.
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  two days are equivalent up to addition/subtraction of multiple of 7.
  10 days from now is day 5 again, Friday!

What day is it a year from now?
 This year is a leap year! So 366 days from now.
 Day 2+366 or day 368.
Smallest representation:
 subtract 7 until smaller than 7.
 divide and get remainder.
 368/7 leaves quotient of 52 and remainder 4.
  or February 6, 2025 is Day 4, a Thursday.

Day of the week.

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What day is it a year from now? on February 6, 2025?
Number days.
0 for Sunday, 1 for Monday, . . . , 6 for Saturday.

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What day is it a year from now?
This year is a leap year! So 366 days from now.
Day 2+366 or day 368.
Smallest representation:
subtract 7 until smaller than 7.
divide and get remainder.
368/7 leaves quotient of 52 and remainder 4.
or February 6, 2025 is Day 4, a Thursday.

Years and years...

80 years from now? February 6, 2104

20 leap years. 366*20 days

60 regular years. 365*60 days

It is day $2 + 366 * 20 + 365 * 60$. Equivalent to?

Hmm.

What is remainder of 366 when dividing by 7? 2.

What is remainder of 365 when dividing by 7? 1

Today is day 2.

Get Day: 2 + 20*2 + 60*1 = 102

Remainder when dividing by 7? 4.

Or February 6, 2104 is Thursday!

Further Simplify Calculation:

20 has remainder 6 when divided by 7.

60 has remainder 4 when divided by 7.

Get Day: 2 + 6*2 + 4*1 = 18.

Or Day 4. February 6, 2104 is Thursday.

“Reduce” at any time in calculation!

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x-y)$ is divisible by m.
...or $x = y + km$ for some integer k.

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...or x and y have the same remainder w.r.t. m.

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Mod 7 equivalence classes:

Modular Arithmetic: Basics.

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if and only if $(x - y)$ is divisible by m.
...or $x = y + km$ for some integer k.
...or x and y have the same remainder w.r.t. m.

Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$

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if and only if $(x - y)$ is divisible by m.
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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$

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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

Modular Arithmetic: Basics.

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if and only if $(x-y)$ is divisible by m.
...or $x = y + km$ for some integer k.
...or x and y have the same remainder w.r.t. m.

Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$

Modular Arithmetic: Basics.

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if and only if $(x - y)$ is divisible by m.
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Mod 7 equivalence classes:
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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x-y)$ is divisible by m.
...or $x = y + km$ for some integer k.
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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
...or $x = y + km$ for some integer k.
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Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
if and only if $(x - y)$ is divisible by m.
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...or x and y have the same remainder w.r.t. m.

Mod 7 equivalence classes:
$\{\ldots, -7, 0, 7, 14, \ldots\}$ $\{\ldots, -6, 1, 8, 15, \ldots\}$ ...

Useful Fact: Addition, subtraction, multiplication can be done with any equivalent x and y.

or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore,

Modular Arithmetic: Basics.

x **is congruent to** y **modulo** m or "$x \equiv y \pmod{m}$"
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...or $x = y + km$ for some integer k.
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Mod 7 equivalence classes:
$\{\ldots,-7,0,7,14,\ldots\}$ $\{\ldots,-6,1,8,15,\ldots\}$ ...

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or " $a \equiv c \pmod{m}$ and $b \equiv d \pmod{m}$
$\implies a+b \equiv c+d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c+km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d+jm$ for some integer j.
Therefore, $a+b = c+d+(k+j)m$

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Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a + b = c + d + (k+j)m$ and since $k + j$ is integer.

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Therefore, $a+b = c+d+(k+j)m$ and since $k+j$ is integer.
$\implies a+b \equiv c+d \pmod{m}$.

Modular Arithmetic: Basics.

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Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a + b = c + d + (k + j)m$ and since $k + j$ is integer.
$\implies a + b \equiv c + d \pmod{m}$. $\square$

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$\implies a + b \equiv c + d \pmod{m}$ and $a \cdot b = c \cdot d \pmod{m}$"

Proof: If $a \equiv c \pmod{m}$, then $a = c + km$ for some integer k.
If $b \equiv d \pmod{m}$, then $b = d + jm$ for some integer j.
Therefore, $a + b = c + d + (k + j)m$ and since $k + j$ is integer.
$\implies a + b \equiv c + d \pmod{m}$. $\square$

Can calculate with representative in $\{0,\ldots,m-1\}$.

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$S = \{0(4), 1(4), 2(4), 3(4), 4(4), 5(4)\} = \{0, 4, 8, 12, 16, 20\}$

reducing $\pmod 6$

$S = \{0, 4, 2, 0, 4, 2\}$

Not distinct.

Proof review. Consequence.

Thm: If $\gcd(x,m) = 1$, then x has a multiplicative inverse modulo m.

Proof Sketch: The set $S = \{0x, 1x, \ldots, (m-1)x\}$ contains $y \equiv 1 \mod m$ if all distinct modulo m.

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All distinct, contains 1!

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$5x = 3 \pmod 6$

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$5x=3 \pmod 6$ What is x? Multiply both sides by 5.

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All distinct, contains 1! 5 is multiplicative inverse of 5 $\pmod 6$.

$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.

$x = 15$

Proof review. Consequence.

Thm: If $\gcd(x,m) = 1$, then x has a multiplicative inverse modulo m.

Proof Sketch: The set $S = \{0x, 1x, \ldots, (m-1)x\}$ contains $y \equiv 1 \mod m$ if all distinct modulo m.

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$5x = 3 \pmod{6}$ What is x? Multiply both sides by 5.

$x = 15 = 3 \pmod{6}$

Proof review. Consequence.

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$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.

$x = 15 = 3 \pmod 6$

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$4x = 2 \pmod 6$ Two solutions!

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$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.

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$5x = 3 \pmod 6$ What is x? Multiply both sides by 5.
$x = 15 = 3 \pmod 6$

$4x = 3 \pmod 6$ No solutions. Can't get an odd.
$4x = 2 \pmod 6$ Two solutions! $x = 2, 5 \pmod 6$

Very different for elements with inverses.

Finding inverses.

How to find the inverse?

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x,m).

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x,m).
Greater than 1?

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.
Equal to 1?

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.
Equal to 1? Mutliplicative inverse.

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.
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Algorithm:

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
  Greater than 1? No multiplicative inverse.
  Equal to 1? Mutliplicative inverse.

Algorithm: Try all numbers up to x to see if it divides both x and m.

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
Greater than 1? No multiplicative inverse.
Equal to 1? Mutliplicative inverse.

Algorithm: Try all numbers up to x to see if it divides both x and m.

Very slow.

Finding inverses.

How to find the inverse?

How to find **if** x has an inverse modulo m?

Find gcd (x, m).
 Greater than 1? No multiplicative inverse.
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Very slow.

Next: A Faster algorithm.